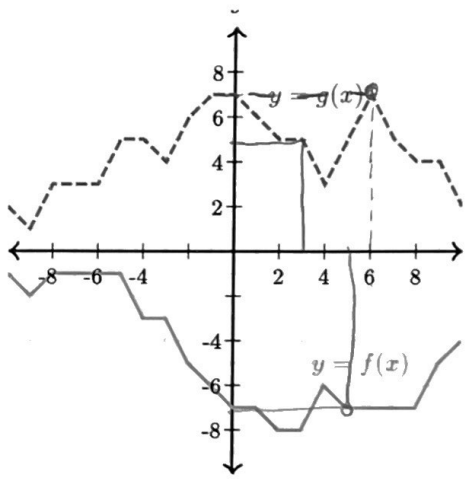
Part 1: Calculus Readiness Quiz

We will have a 'readiness' quiz during the first two weeks of school on this material. The quiz will be a great indicator of whether or not you will feel prepared to take this course!

The practice problems below are similar to problems you could see on the quiz. The formatting is organized so if you need extra practice, you can find it on Khan Academy. Each section matches a section from the course called 'Getting Ready for AP Calculus'.

Getting Ready for Limits and Continuity (Unit 1)

Functions

$q(x) = -x^2 + 2x$	
$r(x) = \frac{5}{x}$	
$h(x) = \sqrt{3x - 5}$	
$p(x) = \begin{cases} 3 - 5x & x \leq 3 \\ 7 & 3 < x \leq 7 \\ \frac{x^2+4}{x} & x > 7 \end{cases}$	

1. Evaluate each of the following:

   a. $q(4) = -(4)^2 + 2(4) = -16 + 8 = \boxed{-8}$

   b. $h(4) = \sqrt{3(4) - 5} = \sqrt{12 - 5} = \boxed{\sqrt{7}}$

   c. $g(6)$ go to x=6, find what output/y value matches on g(x) graph $\boxed{7}$

   d. $r(0)$ = undefined b/c you can't ÷ by 0

2. Evaluate each of the following:

   a. $p(0)$ $0 \leq 3$ so $p(0) = 3 - 5(0) = \boxed{3}$

   b. $p(4)$ $3 < 4 \leq 7$ so $p(4) = \boxed{7}$

   c. $p(7)$ $3 < 7 \leq 7$ so $p(7) = \boxed{7}$

3. Evaluate each of the following:

   a. $r(h(35))$ $h(35) = \sqrt{3(35) - 5} = 10$

   $r(10) = \frac{5}{10} = \boxed{\frac{1}{2}}$

   b. $f(g(3))$

   $g(3) = 5$ $f(5) = \boxed{-5}$

   c. $q(h(x)) = -\left(\sqrt{3x-5}\right)^2 + 2\sqrt{3x-5}$

   $-(3x-5) + 2\sqrt{3x-5}$

   $\boxed{-3x + 5 + 2\sqrt{3x-5}}$

Factoring

4. Factor each completely:

a. $x^2 - 11x + 30$

$(x-5)(x-6)$

b. $4x^2 + 8x - 32$

$4(x^2+2x-8)$

$4(x-2)(x+4)$

c. $3x^3 + 2x^2 - 12x - 8$

$(3x^3+2x^2)+(-12x-8)$

$x^2(3x+2)-4(3x+2)$

$(x^2-4)(3x+2) \rightarrow (x-2)(x+2)(3x+2)$

d. $16x^4 - 81$

$(4x^2-9)(4x^2+9)$

$(2x-3)(2x+3)(4x^2+9)$

e. $3x^2 + 7x + 4$

$(3x+4)(x+1)$

Trig

5. Evaluate the following: (you may need to use the unit circle on the last page)

a. $sin(\frac{\pi}{4}) = Y = \sqrt{2}/2$

b. $cos(\frac{4\pi}{3}) = X = -\frac{1}{2}$

c. $tan(\pi) = \frac{Y}{X} = \frac{0}{-1} = 0$

(-1,0)

d. $sec(\frac{\pi}{2}) = \frac{1}{X} = \frac{1}{0} =$ undefined

e. $csc(\frac{7\pi}{4}) = \frac{1}{Y} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

f. $cos(\pi) = X = -1$

6. Simplify using trig identities so there is no fraction.

a. $\frac{1-sin^2x}{cos\,x} = \frac{cos^2x}{cos\,x} = cos\,x$

b. $\frac{sin\,x}{cos\,x} = tan\,x$

c. $\frac{cos\,x}{sec\,x} = \frac{cos\,x}{\frac{1}{cos\,x}} = cos\,x \cdot cos\,x = cos^2x$

d. $\frac{sec^2x-1}{sin\,x} = \frac{cos^2x}{sin\,x} = \frac{sin\,x}{cos\,x} \cdot \frac{1}{sin\,x} = \frac{1}{cos\,x} = sec\,x$

Rational Expressions

7. Simplify the following. Make sure there are no radicals in the denominator.

a. $\frac{x}{x+3}+\frac{1}{x}$

$\frac{x^2}{x(x+3)}+\frac{x+3}{x(x+3)}=\boxed{\frac{x^2+x+3}{x(x+3)}}$

b. $\frac{x^2+3}{x^3}\div\frac{x+1}{x^4}$

$\frac{x^2+3}{x^3}\cdot\frac{x^4}{x+1}=\boxed{\frac{x(x^2+3)}{x+1}}$

c. $\frac{\frac{1}{x}+1}{\frac{1}{x}-1}=\frac{\frac{1+x}{x}}{\frac{1-x}{x}}=\frac{1+x}{x}\cdot\frac{x}{1-x}$

$=\boxed{\frac{1+x}{1-x}}$

d. $\frac{3}{\sqrt{x}+1}\cdot\frac{\sqrt{x}-1}{\sqrt{x}-1}=\frac{3(\sqrt{x}-1)}{x-1}$

$\boxed{\frac{3(\sqrt{x}-1)}{x-1}}$

8. Simplify the following. Which x values make the original expression undefined?

a. $\frac{15x^2-25x}{3x-5}=\frac{5x(3x-5)}{3x-5}=\boxed{5x}$

DNE when $3x-5=0$

$+5\ +5$

$3x=5$

$\boxed{x=5/3}$

b. $\frac{2x^2-14x-36}{x^2-9x}=\frac{2(x^2-7x-18)}{x(x-9)}$

DNE when $\boxed{x=0}$ and $\boxed{x=9}$

$=\frac{2(x-9)(x+2)}{x(x-9)}$

$=\boxed{\frac{2(x+2)}{x}}$

Getting Ready for Differentiation: Definition and Basic Rules (Unit 2)

Equations of Lines $y-y_1=m(x-x_1)$

9. Create a linear equation for the given information/representation:

a. Through the point $(\overset{x}{2},\overset{y}{1})$ with slope 4.

$\boxed{y-1=4(x-2)}$

b. Through points $(\overset{x}{4},\overset{y}{10})$ and $(\overset{x}{6},\overset{y}{10})$.

$m=\frac{10-10}{4-6}=\frac{0}{-2}$

$\boxed{y=10}$

$m = \frac{7-5}{2-4} = \frac{2}{-2} = -1$

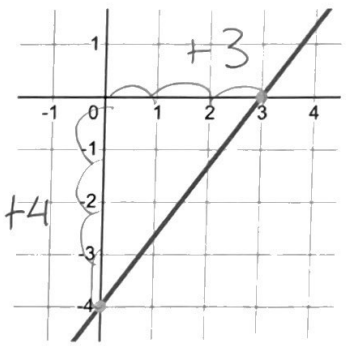
c. Through points $(2, 7)$ and $(4, 5)$ (x y, x y)

$y - 7 = -1(x-2)$

d. Through the point (5, 9) with an undefined slope.

$x = 5$

e. For the graph below:



$m = \frac{4}{3}$

$y = \frac{4}{3}x - 4$

Exponents and Logs

10. Simplify completely so there are no fractions:

a. $\frac{14}{2x} = 7x^{-1}$

b. $\frac{2a^2 \cdot a^4}{a^5} = \frac{2a^6}{a^5} = 2a$

11. Evaluate each logarithm without a calculator:

a. $log_9 81 = x \rightarrow 9^x = 81$

$x = 2$

b. $log_{27} 3 = x \rightarrow 27^x = 3$

$\sqrt[3]{27} = 3^1$ so $x = \frac{1}{3}$

c. $log_2 \frac{1}{32} \rightarrow 2^x = \frac{1}{32}$

$x = -5$

Getting Ready for Differentiation: Composite, Implicit, and Inverse Functions (Unit 3)

Literal Equations

12. Solve for y:

a. $x = \frac{2y}{y+3}$ $\cdot (y+3)$

$(y+3)(x) = 2y$

$yx + 3x = 2y$

$-yx \quad -yx$

$3x = 2y - yx$

$3x = y(2-x)$

$y = \frac{3x}{2-x}$

b. $3x + 4y^2 = 19$

$-3x \quad -3x$

$4y^2 = 19 - 3x$

$\sqrt{y^2} = \sqrt{\frac{19-3x}{4}}$

$y = \sqrt{\frac{19-3x}{4}}$

Inverse Functions

13. Find inverse of each of the following:

a. $g(x) = 2x + 1$

$y = 2x + 1$

$x = 2y + 1$

$-1 \qquad -1$

$x - 1 = 2y$

$y = \frac{x-1}{2}$

b. $h(x) = \frac{x^3}{3}$

$y = \frac{x^3}{3}$

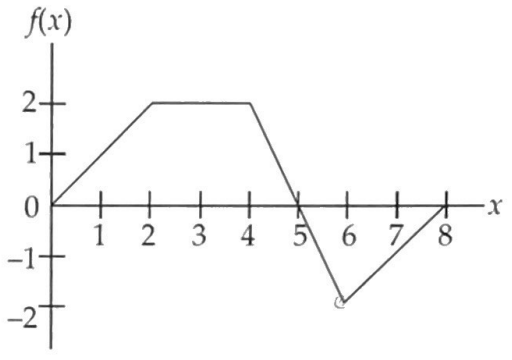
$3 \cdot x = \frac{y^3}{3} \cdot 3$

$y^3 = 3x$

$y = \sqrt[3]{3x}$

14. Given the following representations, find the indicated inverses:

x	-3	4	7	10	13
g(x)	7	2	-3	0	9



a. $g^{-1}(-3)$ where does $g(x) = -3$?

$x = 7$

so $g^{-1}(-3) = 7$

b. $f^{-1}(-2)$

6

What is the x value when $y = -2$?

Get ready for contextual applications of differentiation (Unit 4)

Word Problems and Modeling

15. Avis car rental charges a flat rental fee, plus an additional 10 cents per mile driven. When I rented a car with them, I drove 150 miles and paid $330. Write a linear equation to represent the relationship between the amount of money you are charged (y) and how many miles you drive (x).

(150 miles, $330) m = slope = .10

$y - 330 = .1(x - 150)$

(0, 7.5) (5, 4.45)

16. A kiddie pool has sprung a leak. Yikes. It starts with 7.5 gal of water. Five minutes after the leak began, there was 4.45 gallons left in the kiddie pool. Write a linear equation to represent the gallons left in the pool (y) and how much time has passed since the leak started (x).

$m = \text{Slope} = \frac{7.5-4.45}{0-5} = \frac{3.05}{-5} = -.61$ $\quad y - 0 = -.61(x - 7.5)$

$y = -.61(x - 7.5)$

17. Lennox owns a big apple orchard. She ships her apples to various markets using a fleet of trucks. Every week, each truck goes on 3 trips, and for each trip Lennox gets 300 dollars. On a single trip, a truck delivers 50 packs, and each pack contains 12 kilograms of apples. Overall, Lennox sells 4500 dollars worth of apples in a week. How many trucks does she have?

This has info we don't need!

1 Truck = 3 trips x $300/trip = $900

$\frac{4500}{900} =$ 5 trucks

18. Jesse is filling spherical balloons. When full, one of these balloons has a diameter of 24cm. Jesse can fill a balloon at a rate of 820 cm^3 per breath. How many breaths will it take for Jesse to fill a balloon?

Volume $= \frac{4}{3}\pi r^3$ $\quad d = 24$ $\quad r = 12$

Volume $= \frac{4}{3}\pi(12)^3 = 7238.23$

$\frac{7238.23\ cm^3}{820\ cm^3/hr} = 8.82$ breaths, so 9 breaths

19. Let $G(t)$ represent the gallons of water in a pool t minutes after it began to drain. Interpret the following:

a. G(3) = 100 at 3 minutes, the pool has 100 gallons of water

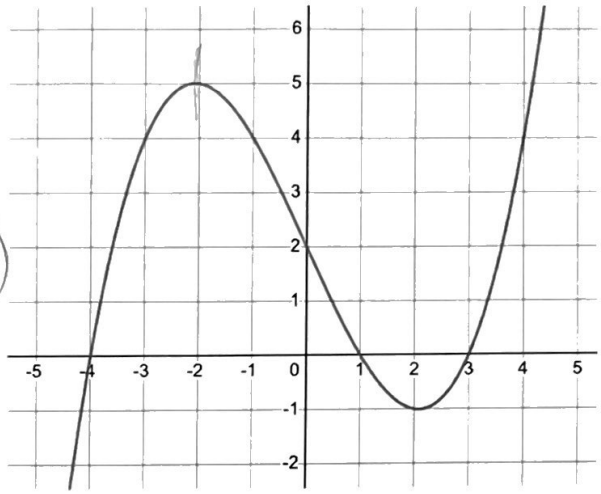
b. G(0) = 120 at the start, the pool has 120 gal of water

c. G(20) > G(25) The pool has more water at 20 mins than at 25 mins

Get ready for applying derivatives to analyze functions (Unit 5)

Interpreting Features of a Graph

20. identify the following: maxima/minima, intervals of increasing/decreasing, zeroes (you can estimate to the nearest half)



Max @ (-2, 5)

Min @ (2, -1)

Zeroes @ x = -4, x = 1, x = 3

Inc on $(-\infty, -2)$ and $(2, \infty)$ interval notation

dec on $(-2, 2)$

Polynomials

21. Find zeros of each function:

a. $f(x) = x^3 + 11x^2 - x - 11$

$0 = x^3 + 11x^2 - x - 11$

$0 = (x^3 + 11x^2) + (-x - 11)$

$0 = x^2(x + 11) - 1(x + 11)$

$0 = (x^2 - 1)(x + 11)$

$0 = (x - 1)(x + 1)(x + 11)$

So, $x = 1$, $x = -1$, $x = -11$

b. $h(x) = x^4 - 6x^2$

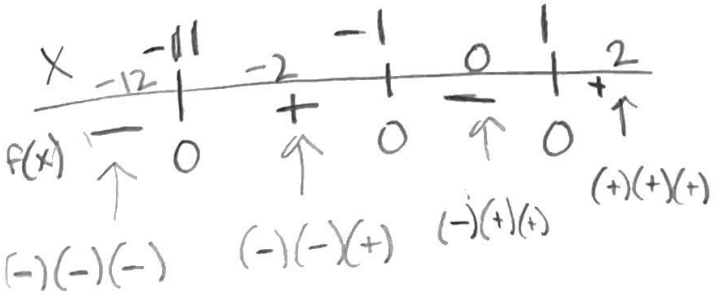
$0 = x^2(x^2 - 6)$

$x^2 = 0$ $\quad$ $x^2 - 6 = 0$

$x = 0$ $\quad$ $x = \pm\sqrt{6}$

22. Based on the zeroes you found above, determine on which intervals the function is positive and which intervals the function is negative. You may write your answer as inequalities or in interval notation. ① set up a table of the zeros ② use test pts

a. $f(x) = x^3 + 11x^2 - x - 11$ $\quad$ b. $h(x) = x^4 - 6x^2$



Positive on $(-11, -1)$ and $(1, \infty)$

Negative on $(-\infty, -11)$ and $(-1, 1)$

Get ready for integration and accumulation of change (Unit 6)

Basic Graphing

23. Sketch a graph of each function.

a. $g(x) = \sqrt{x}$

b. $h(x) = -|x| + 3$

c. $f(x) = (x-2)^2$

See last page :)

d. $p(x) = \sin x$

e. $q(x) = \cos x$

f. $r(x) = e^x$

Get ready for applications of integration (Units 7 and 8)

Solving Equations

24. Solve.

a. $-7(3n+8)+1 = 36-8n$

$-21n - 56 + 1 = 36 - 8n$

$+8n \qquad +8n$

$-13n - 55 = 36$

$+55 \quad +55$

$-13n = 91$

$n = -7$

b. $p^2 + 8p + 20 = 5$

$-5 \quad -5$

$p^2 + 8p + 15 = 0$

$(p+5)(p+3) = 0$

$p = -5$ $p = -3$

c. $4x^3 - 12x = 0$

$4x(x^2 - 3) = 0$

$4x = 0 \qquad x^2 - 3 = 0$

$x = 0 \qquad x^2 = 3$

$x = \pm\sqrt{3}$

25. Solve. Be sure to check for extraneous solutions:

a. $\sqrt{-6-n}^2 = \sqrt{-16-2n}^2$

$-6 - n = -16 - 2n$

$+2n \qquad +2n$

$-6 + n = -16$

$+6 \qquad +6$

$n = -10$

Check: $\sqrt{-6-(-10)} = \sqrt{-16-2(-10)}$

$\sqrt{4} = \sqrt{4}$ ✓

b. $\sqrt{34-3p}^2 = (p-8)^2$

$34 - 3p = p^2 - 16p + 64$

$-34 + 3p \qquad +3p - 34$

$0 = p^2 - 13p + 30$

$0 = (p-10)(p-3)$

$p = 10$ ~~$p = 3$~~

Check: $\sqrt{34-3(10)} = \sqrt{4} = 2 = 10-8$ ✓

$\sqrt{34-3(3)} = \sqrt{25} = 5 \neq 3-8$

c. $(n-2)^{3/4} - 7 = 20$

$+7 \quad +7$

$(n-2)^{3/4} = 27$

$\left((n-2)^{3/4}\right)^{4/3} = 27^{4/3}$

$n-2 = \left(\sqrt[3]{27}\right)^4$

$n-2 = 3^4$

$n-2 = 81$

$n = 83$

d. $\frac{-2}{x+3} = \frac{1}{x+1}$ $\quad \times(x+3) \quad \times(x+1)$

$\frac{-2(x+1)}{x+3} = 1(x+3)$

$-2x-2 = x+3$

$+2x \quad +2x$

$-2 = 3x+3$

$-3 \quad -3$

$-5 = 3x$

$-5/3 = x$

26. Solve.

a. $12 = 4^x$

$\ln 12 = \ln 4^x$

$\frac{\ln 12}{\ln 4} = \frac{x \ln 4}{\ln 4}$

$x = \frac{\ln 12}{\ln 4} = 1.792$

b. $4^{n-2} + 6 = 36$

$-6 \quad -6$

$4^{n-2} = 30$

$\ln 4^{n-2} = \ln 30$

$\frac{(n-2)(\ln 4)}{\ln 4} = \frac{\ln 30}{\ln 4}$

$n-2 = 2.453$

$+2 \quad +2$

$n = 4.453$

c. $\frac{124}{3} = \frac{3e^{2x}}{3}$

$41.333 = e^{2x}$

$\ln(41.333) = \ln e^{2x}$

$\frac{\ln(41.333)}{2} = \frac{2x}{2}$

$1.861 = x$

ln and e cancel each other out!

27. Solve each system.

a. $x + y = 13$ $\quad \cdot -2$

$2x - 4y = 14$

$-2x - 2y = -26$

$+ \; 2x - 4y = 14$

$-6y = -12$

$y = 2$

$x + y = 13$

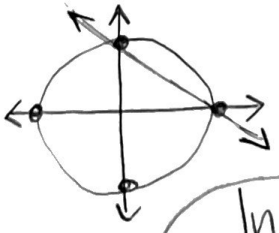
$x + 2 = 13$

$x = 11$

$(11, 2)$

b. $x^2 + y^2 = 25$ ← circle w/ radius 5

$y = x + 5$ ← line w/ slope $m = 1$, y intercept 5



Intersect at $(0,5)$ and $(5,0)$

28. Solve: (write your answer in radians, your answers should be on the interval $0 \leq \theta \leq 2\pi$)

a. $-6\sin\theta = 3$

$\sin\theta = \frac{3}{-6}$

$\sin\theta = -\frac{1}{2}$

Where is the y value $= -1/2$?

$\theta = 7\pi/6$

$\theta = 11\pi/6$

Sin is y value on unit circle

b. $2\cos\theta + \sqrt{3} = 0$

$2\cos\theta = -\sqrt{3}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = 5\pi/6$

$\theta = 7\pi/6$

c. $\cos^2\theta = \cos\theta$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$

$\theta = \pi/2$

$\theta = 3\pi/2$

$\cos\theta - 1 = 0$

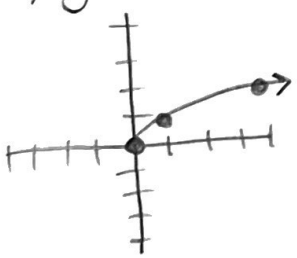
$\cos\theta = 1$

$\theta = 0$

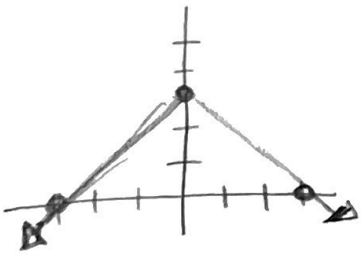
Cos is x value on unit circle

#23 Graphing

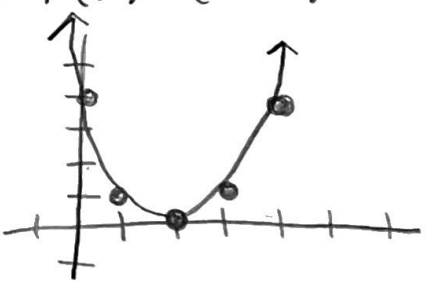
a) $g(x) = \sqrt{x}$



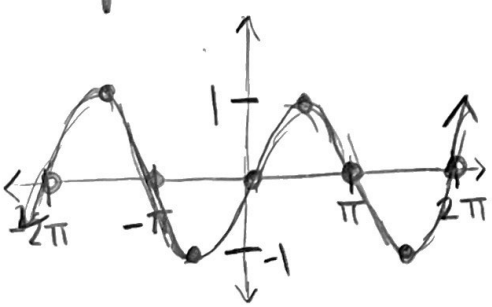
b) $h(x) = -|x| + 3$



c) $f(x) = (x-2)^2$



d) $p(x) = \sin x$



e) $q(x) = \cos x$